Accurate Long-Term Deflection Prediction In Flat Slabs Using Linear Elastic Global Analysis

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Sustaining Infrastructure

Calculating Deflections

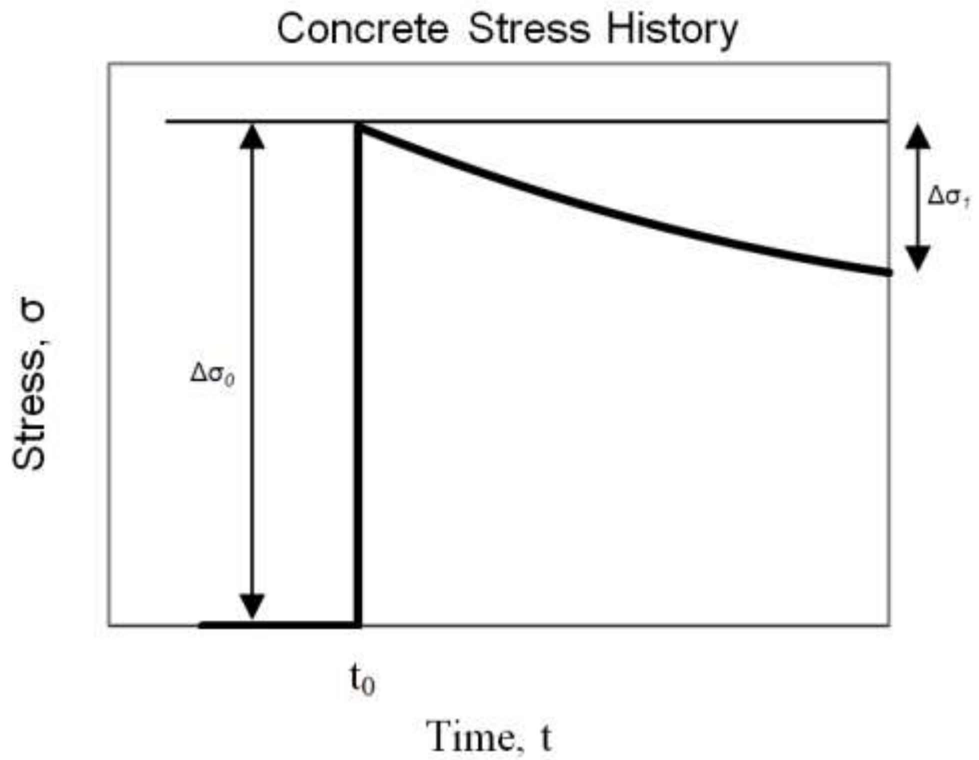
- Thickness limits (deemed to comply)
- Elastic Deflection Multipliers
- 2D Frame Programs
  - Detailed section analysis (including long-term effects)
  - Still not great for irregular plates
- 3D FEA Programs
  - Linear Elastic Finite Element Analysis
  - Non-linear Finite Element Analysis

Factors influencing long-term deflections

- Creep
- Cracking
- Tension Stiffening
- Load History
- Shrinkage and Thermal
  - Externally restrained
  - Internally restrained

Creep

- Use ageing coefficient for gradually induced strains



$$\varepsilon_{LT}(t, t_0) = \Delta\varepsilon_0[1 + \varphi(t, t_0)] + \Delta\varepsilon_1[1 + \chi\varphi(t, t_0)]$$

Cracking and Tension Stiffening

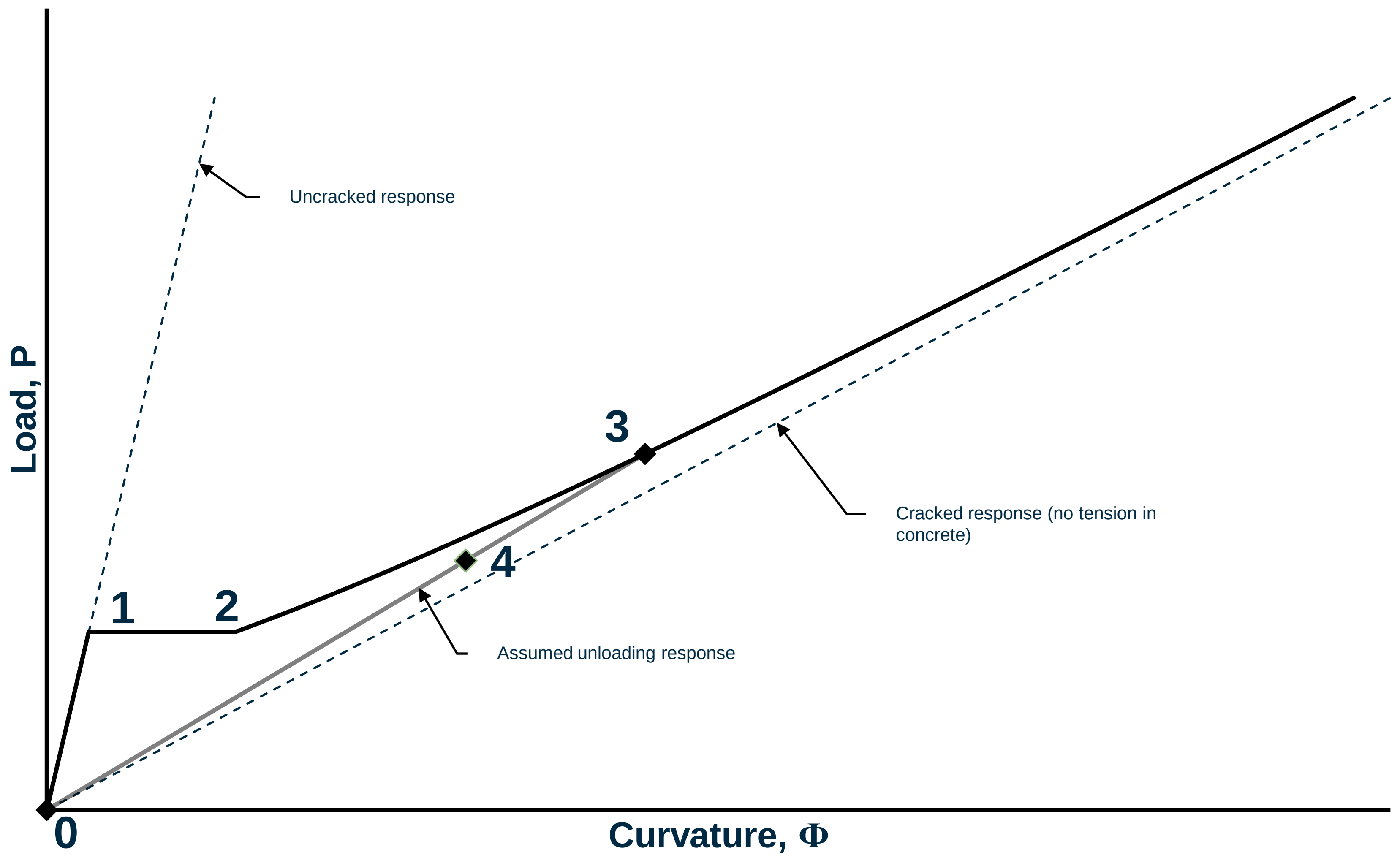
- Eurocode 2 Tension Stiffening Model:

$$\alpha = \zeta \alpha_{11} + (1 - \zeta)\alpha_1$$

$$\zeta = 1 - \beta \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

- Reinforcement stresses consider the effects of shrinkage, etc.

Cracking and Tension Stiffening



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The General Approach

- Perform a linear elastic global analysis and integrate cross section forces
- Perform detailed non-linear long-term calculations on each cross section
- Using resulting axial strains and curvatures, adjust the stiffness of each element tributary to the cross section
- Iterate until solution converges
- Repeat for each load/time stage, breaking each into an instantaneous component (with section force change) and a sustained component (no section force change)

Cross section calculations

- Detailed long-term calculations performed on the cross section
  - Select material stress strain curves
  - Select a tension stiffening model
  - Select creep and shrinkage models

$$\varepsilon_{LT}(t,t_0) = \sum_{i=0}^{n} \Delta\varepsilon_i [1 + \chi\varphi(t,t_i)] + \varepsilon_{sh}(t,t_0)$$

$$\Delta\varepsilon_j = \frac{1}{1+\chi\varphi(t,t_j)} \left\{ \varepsilon_{LT}(t,t_0) - \varepsilon_{sh}(t,t_0) - \sum_{i=0}^{j-1} \Delta\varepsilon_i [1 + \chi\varphi(t,t_i)] \right\}$$

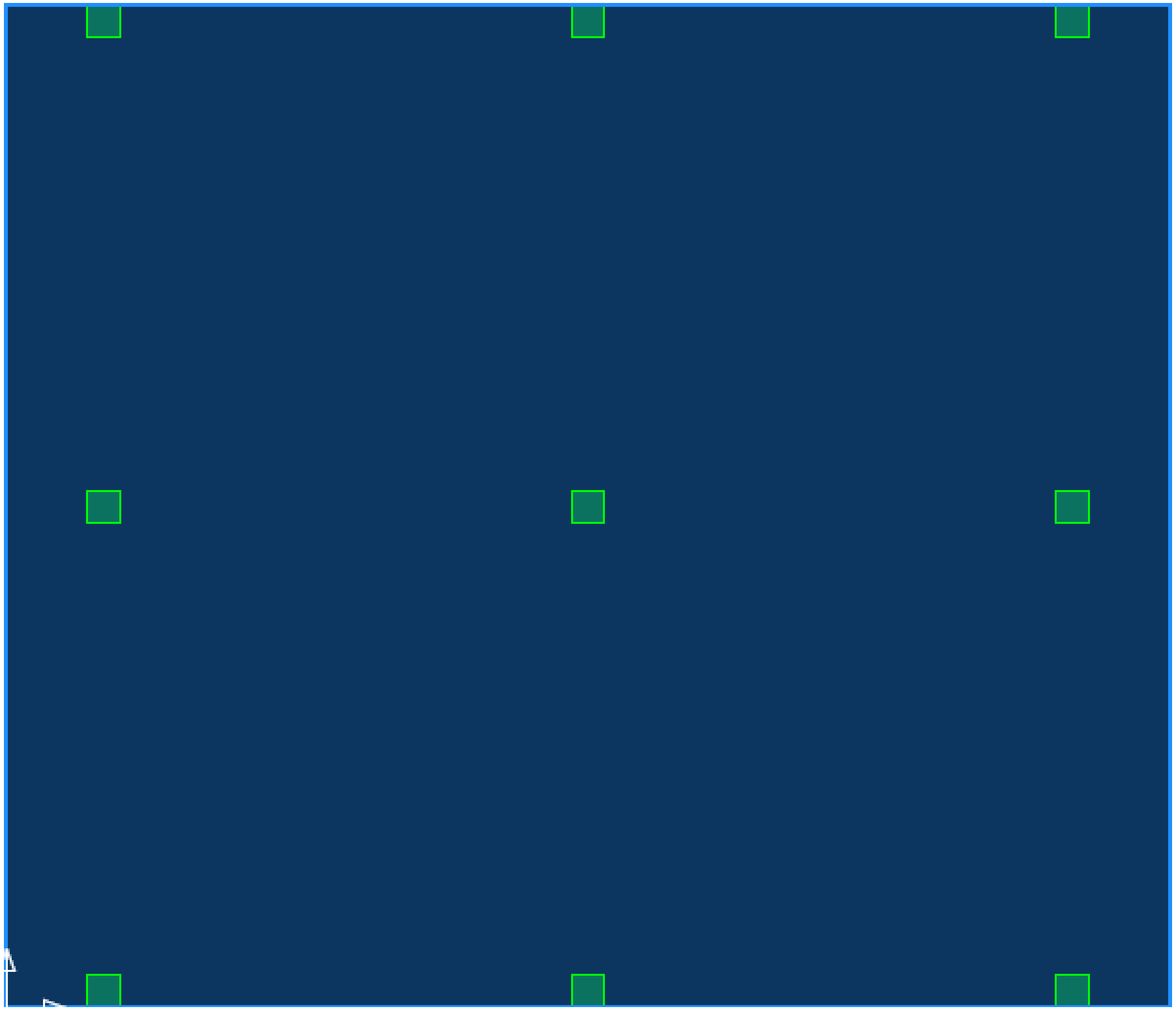
Computer Program Implementation

- Based upon the approach developed
- Uses EC2 tension stiffening model (with modifications/extensions)
- Assumes parabolic concrete stress strain curve and elastoplastic reinforcement stress strain curve
- Uses ACI 209 creep and shrinkage models, user defined total shrinkage strain and ultimate creep factor

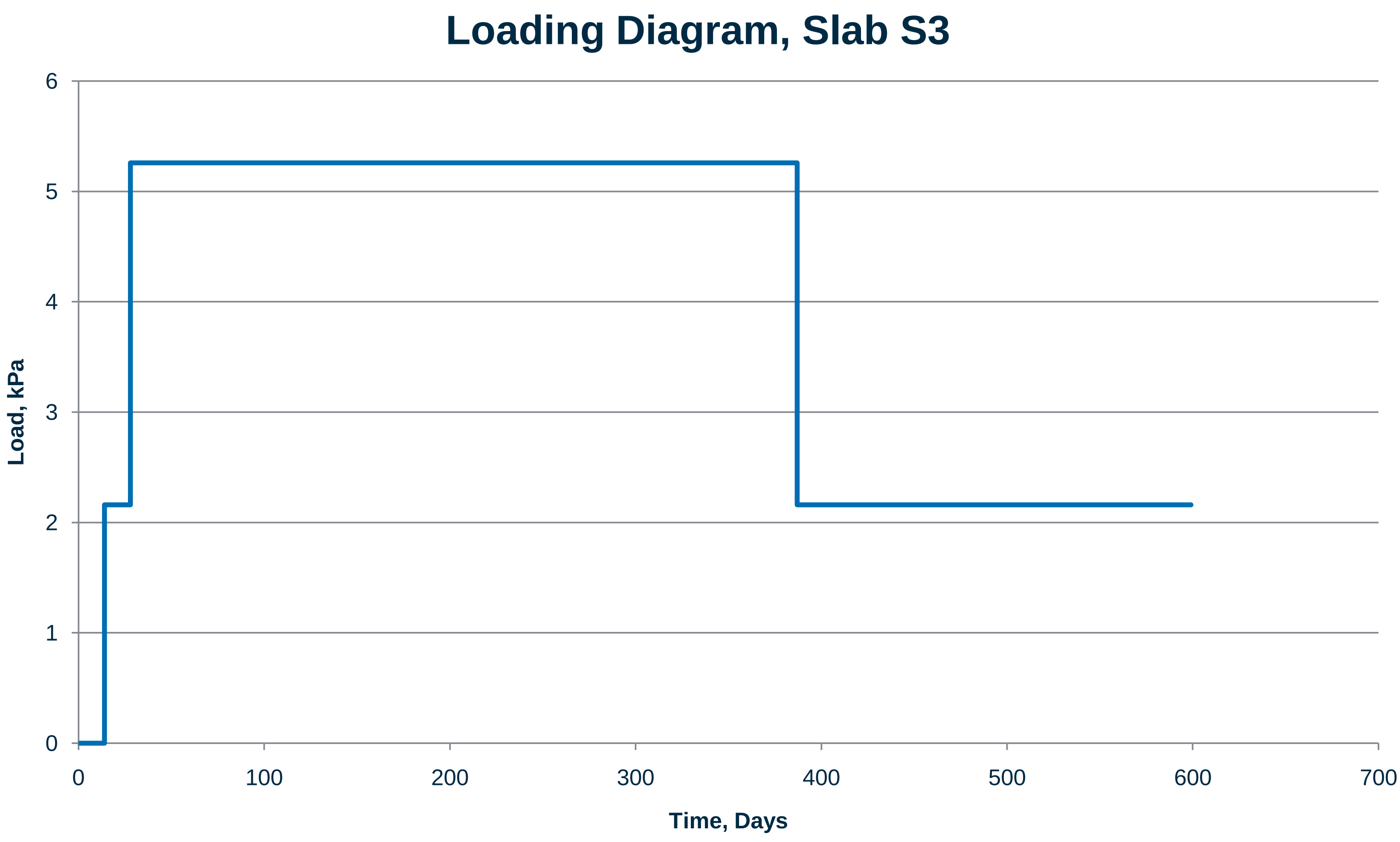
Comparison with Experimental Data

- R.I. Gilbert and X.H Guo, "Time Dependent Deflection and Deformation of Reinforced Concrete Flat Slabs – An Experimental Study", ACI Structural Journal, 102 (3), 2005, pp 363-373
- Tests varied:
  - Concrete strength
  - Column fixity
  - Slab thickness
  - Reinforcement quantities
  - Load magnitude and history
- Analysis models were set up to resemble test properties and material data

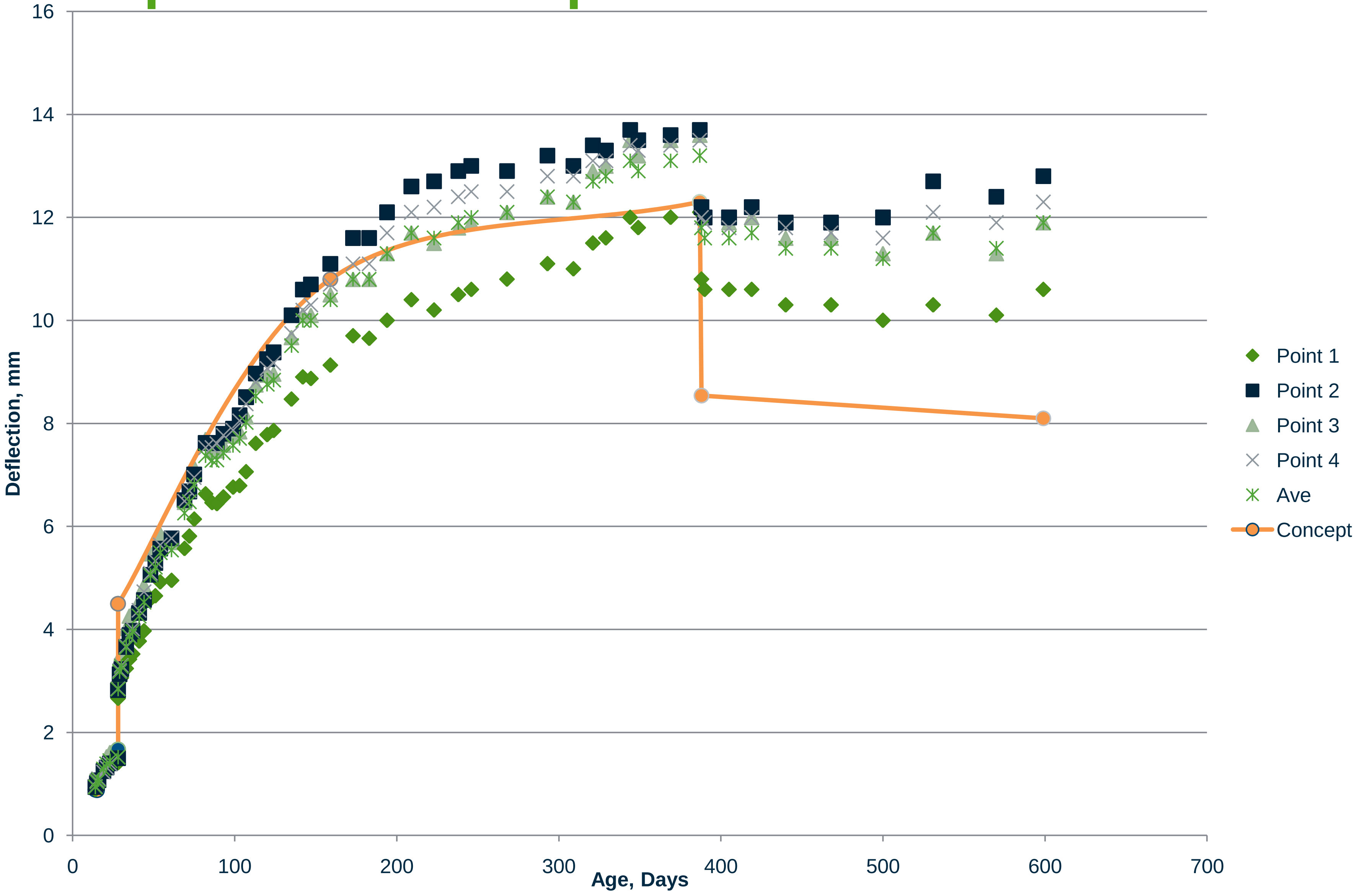
Comparison with Experimental Data



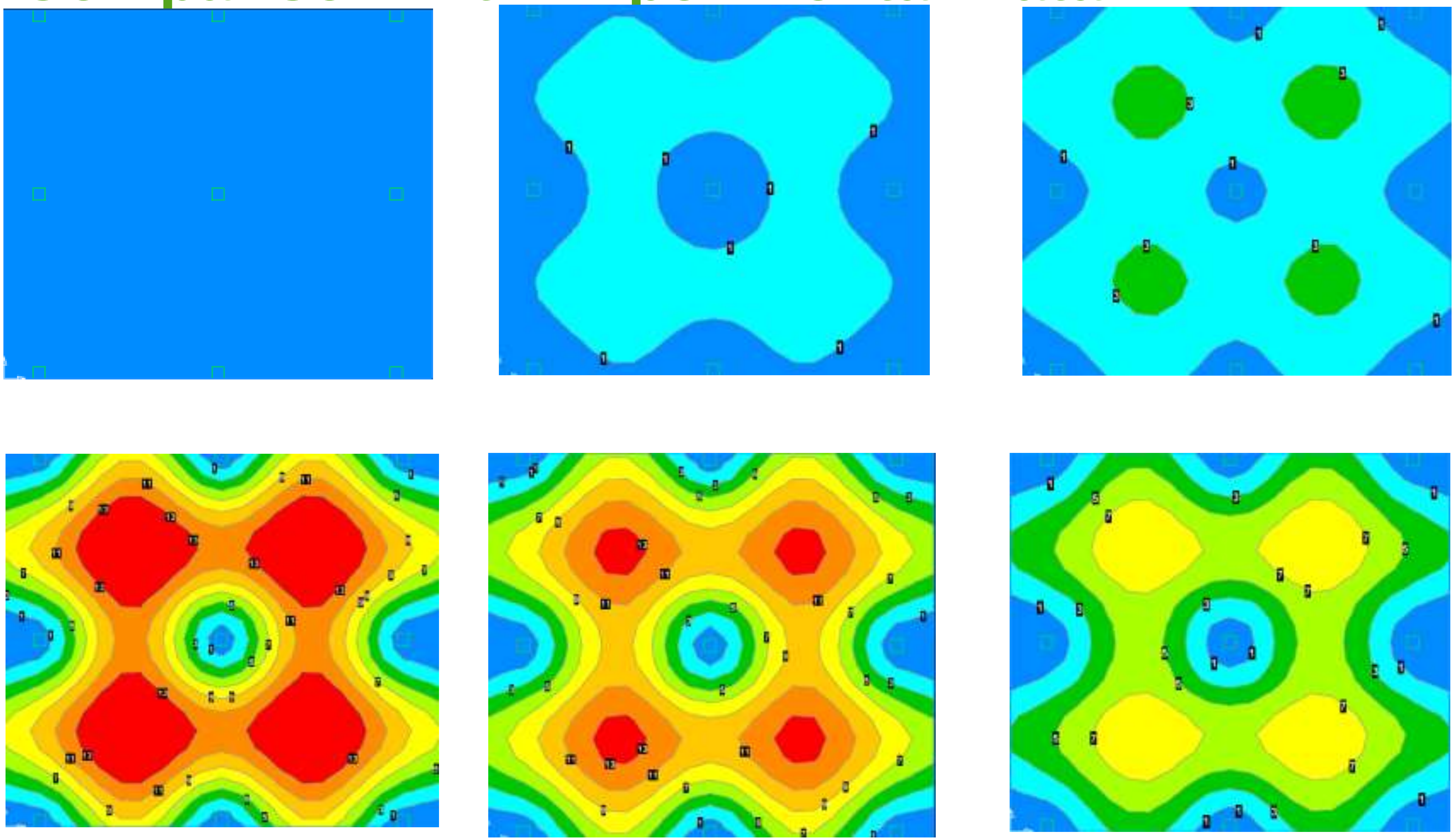
Comparison with Experimental Data



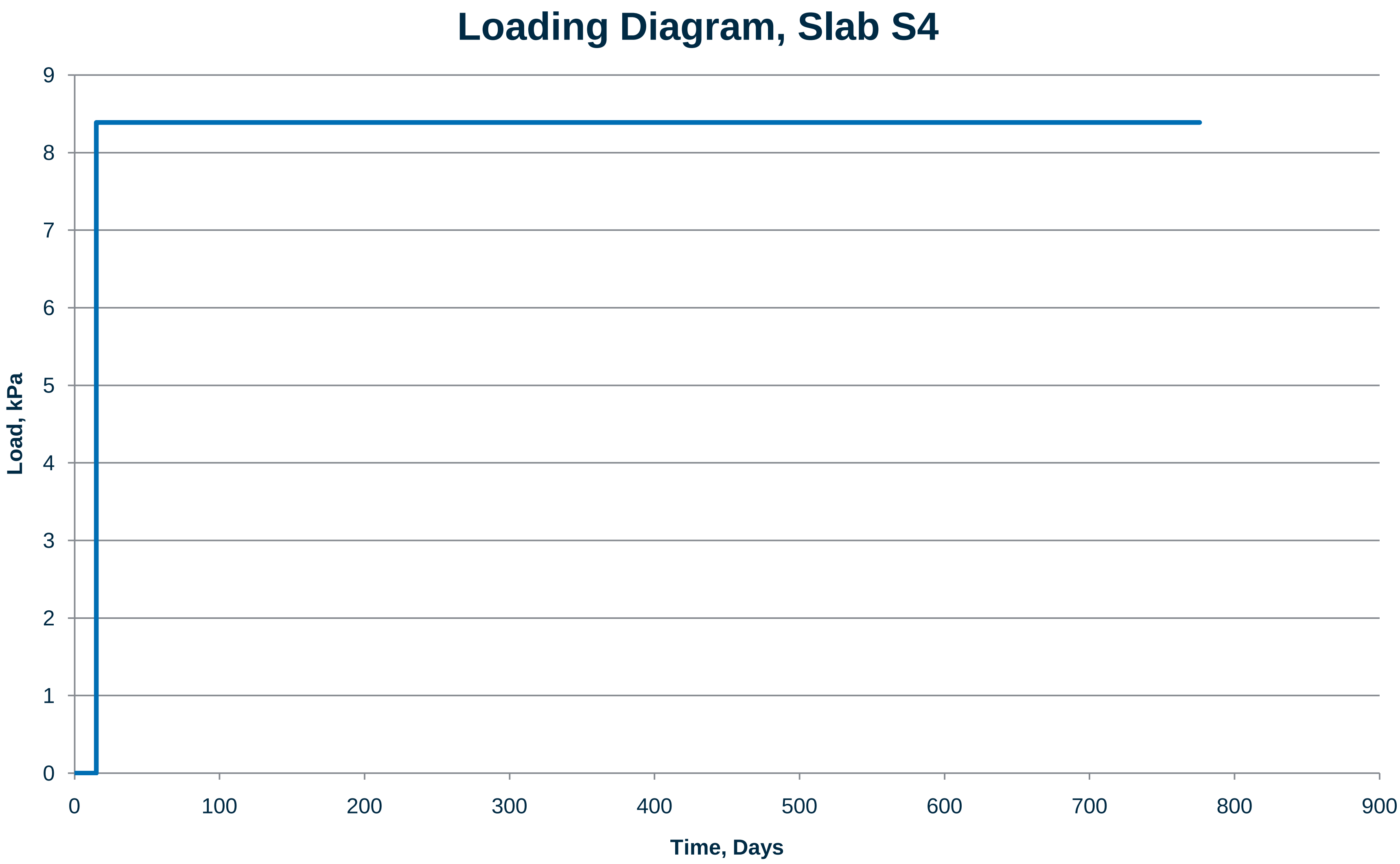
Comparison with Experimental Data – S3



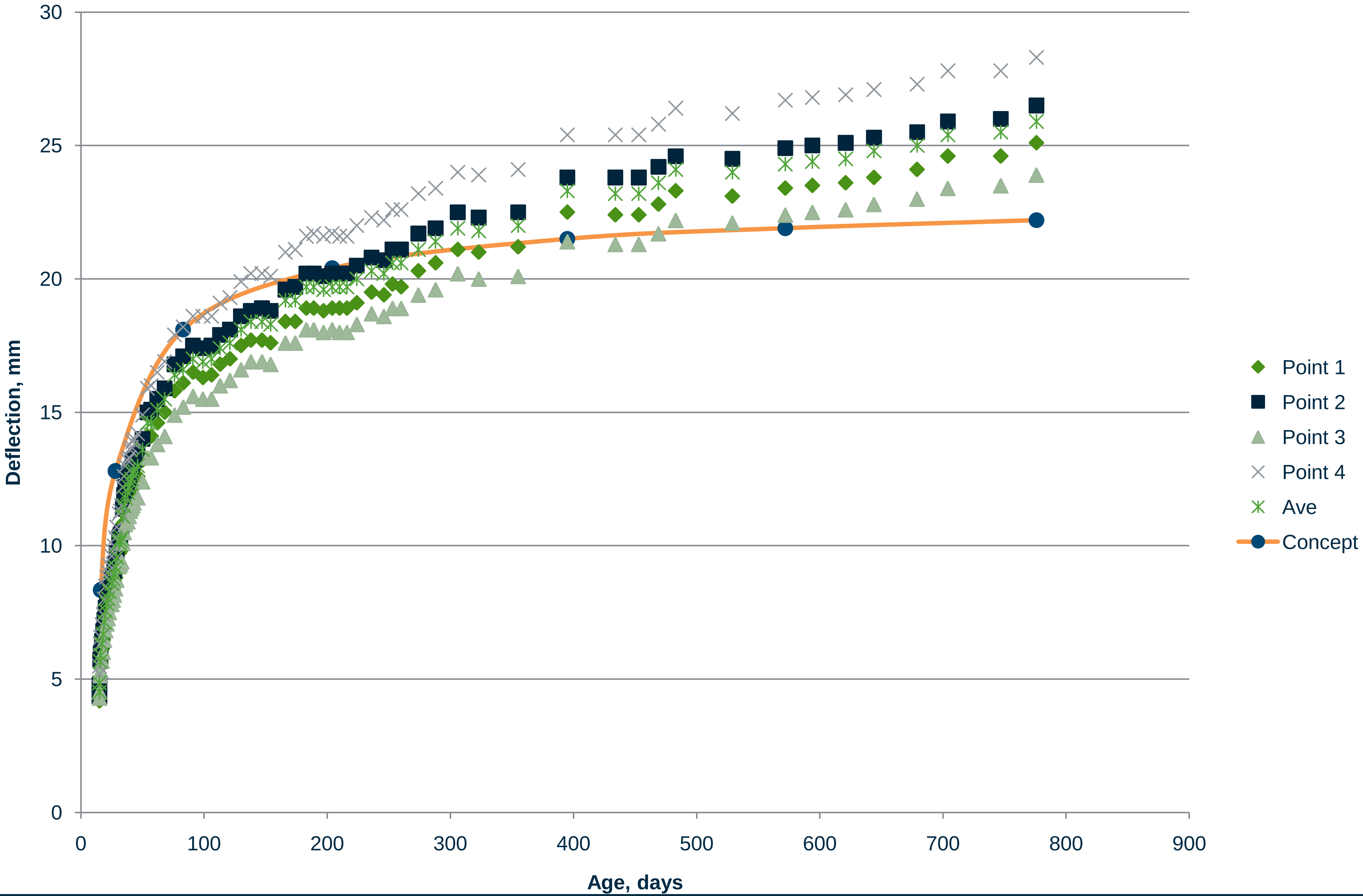
Comparison with Experimental Data



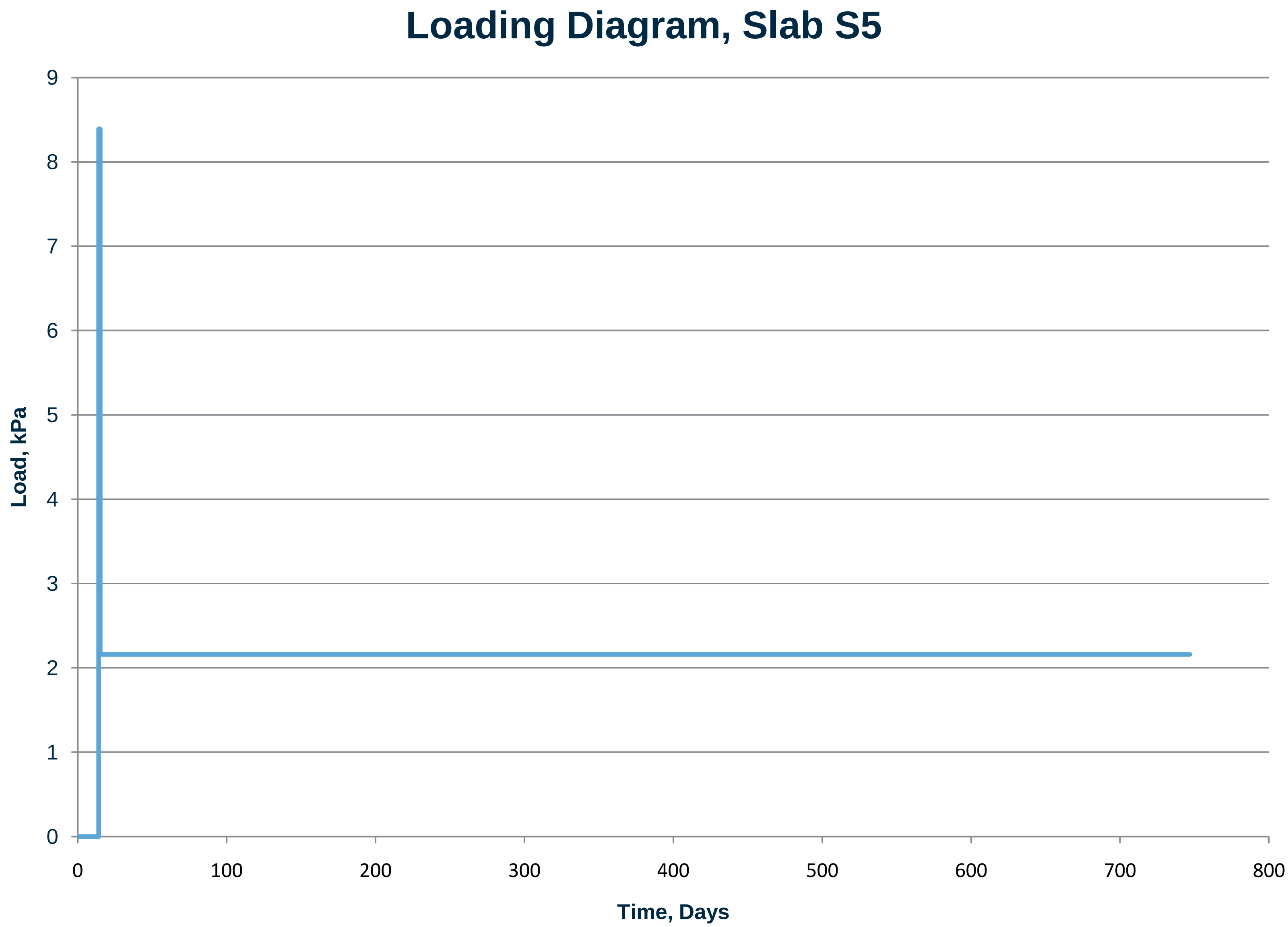
Comparison with Experimental Data



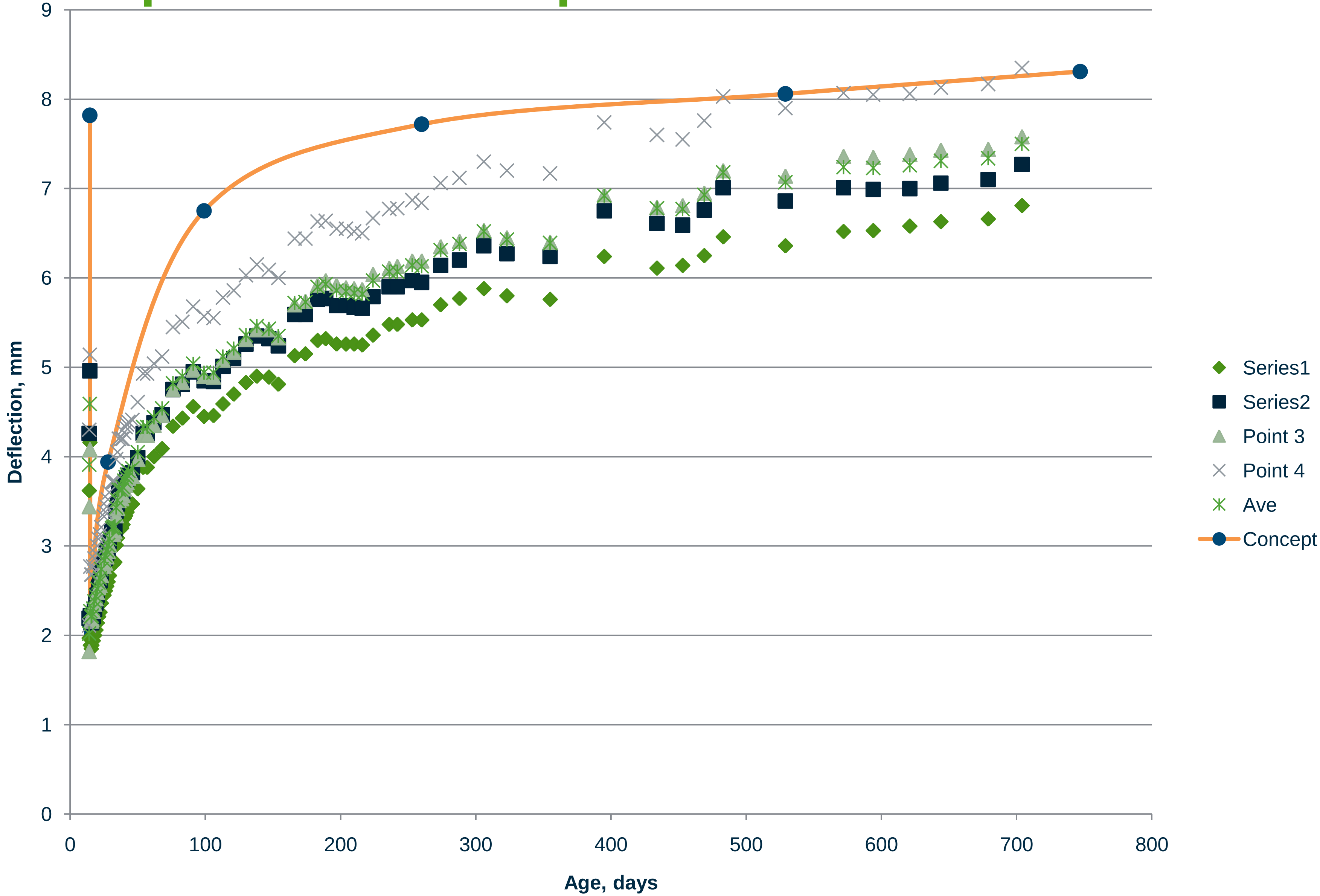
Comparison with Experimental Data – S4



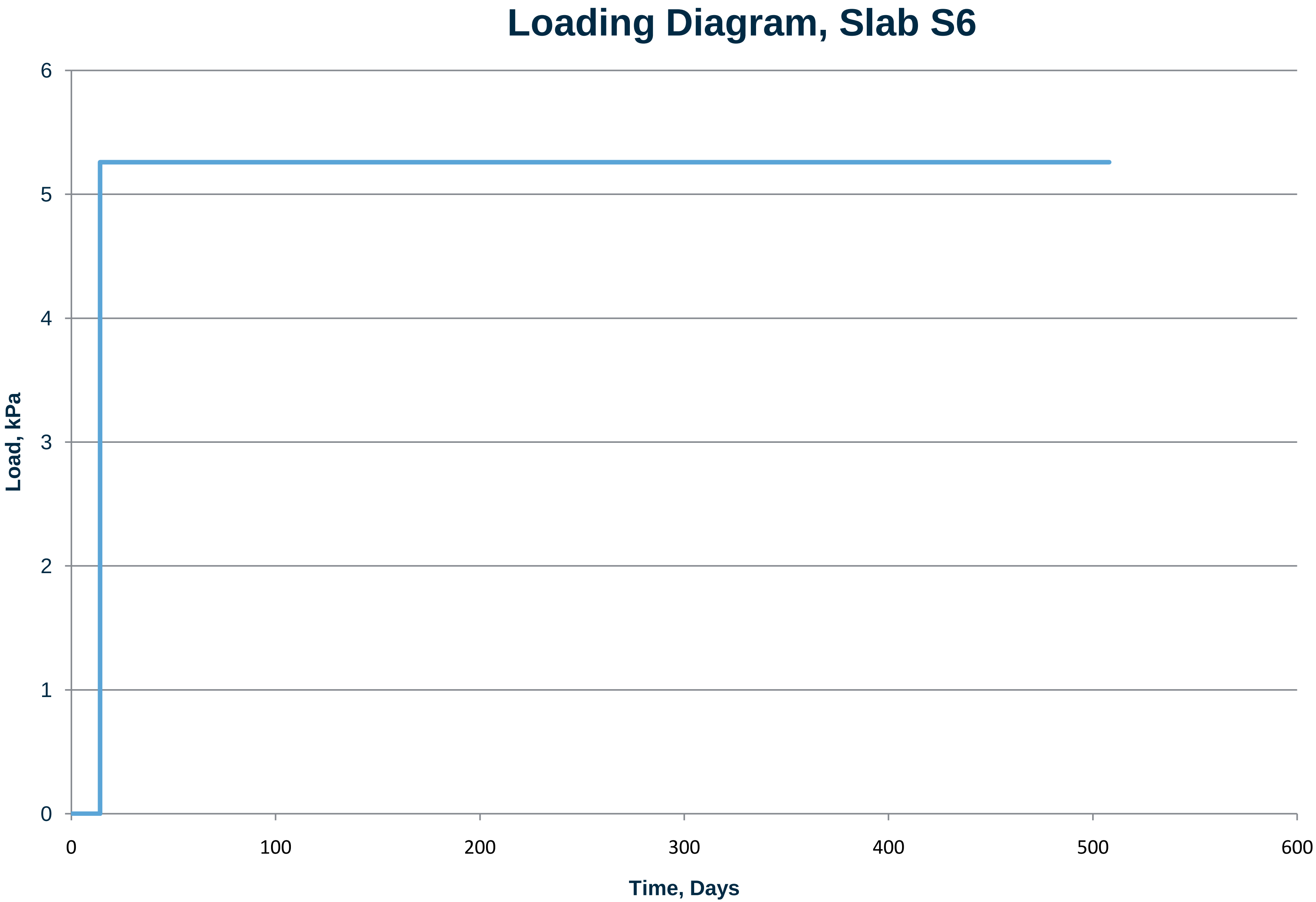
Comparison with Experimental Data



Comparison with Experimental Data – S5



Comparison with Experimental Data



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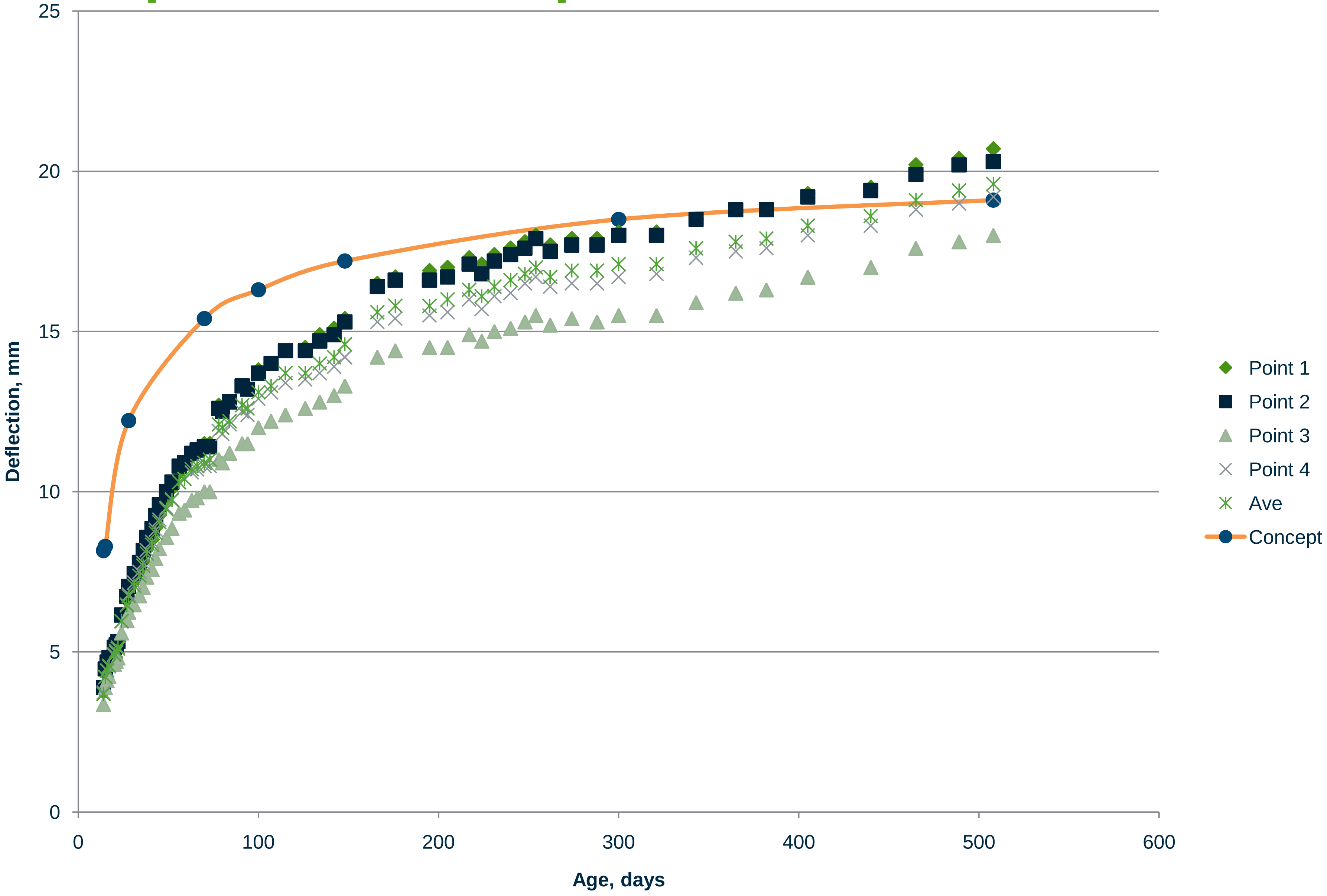
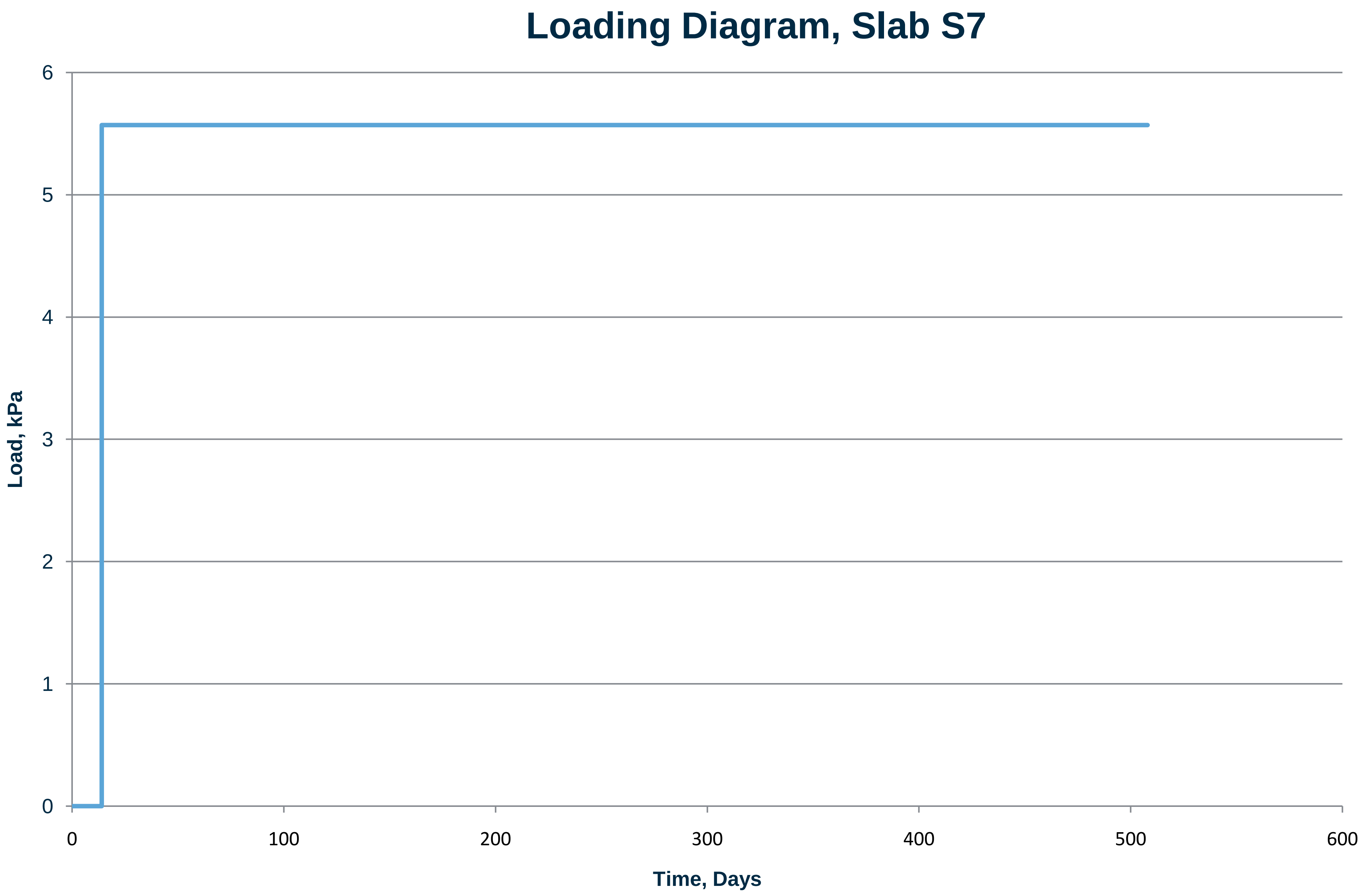
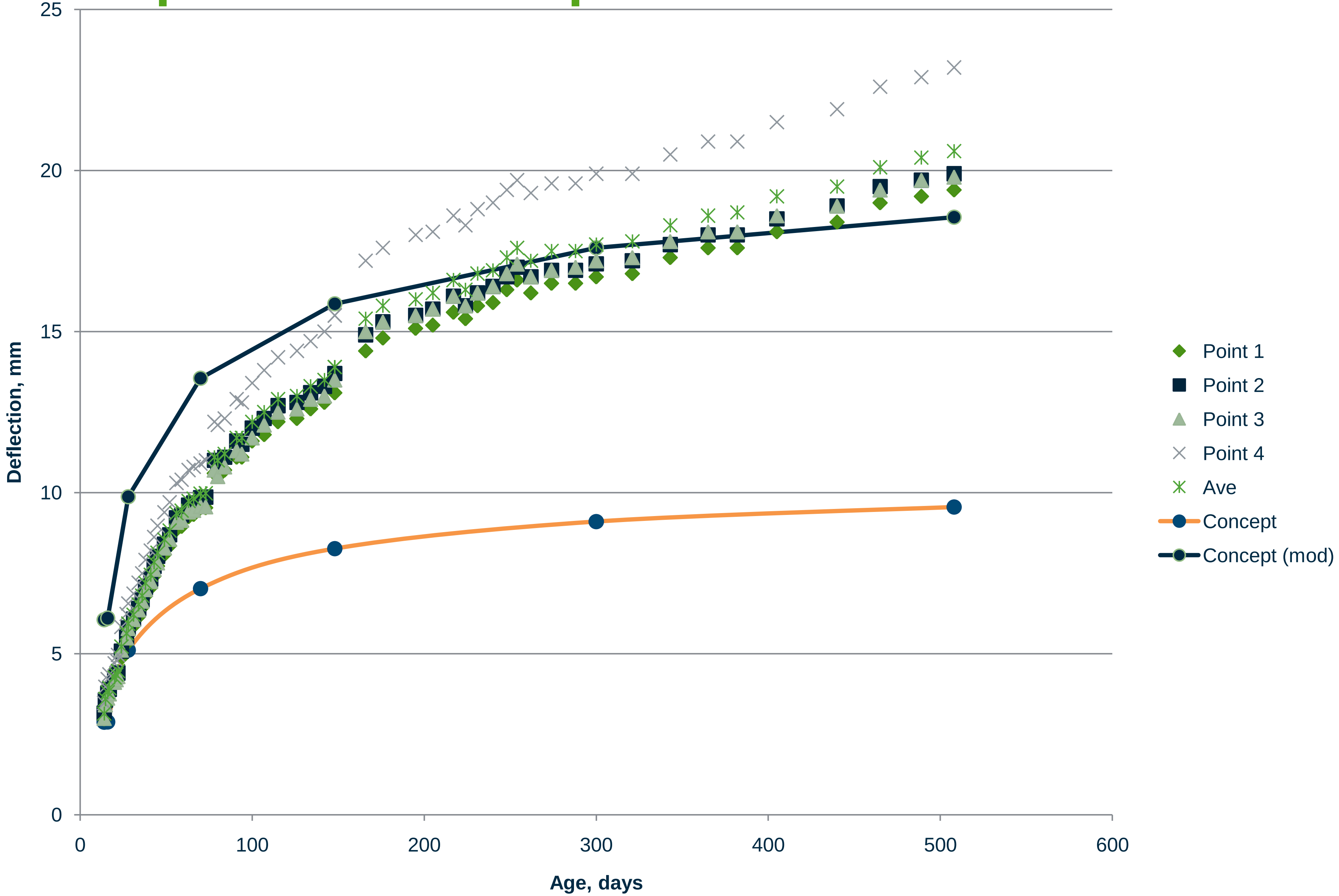
Comparison with Experimental Data – S6



Comparison with Experimental Data



Comparison with Experimental Data – S7



Potential Sources for Discrepancy

- Creep and shrinkage models vary slightly from the test data
- Decision to use long-term beta factor in the tension stiffening model will overestimate early age post cracking results
- Creep recovery overestimated?
- Cannot account for change in sign of curvature due to shrinkage

Conclusions

- An analytical technique was developed accounting for detailed consideration of important long-term factors
  - Applicable to virtually any concrete floor system but works especially well in continuum structures
  - Applies equally as well to reinforced or post-tensioned concrete floors
  - Allows virtually any load history to be specified with results calculated at any point in time
- Comparison with experimental results showed good correlation with test data, except for one case in which the experiment was cracking sensitive