

Twisting Moments in Two-Way Slabs

Design methods for torsion in slabs using finite element analysis

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Section 13.5.1 of ACI 318-08¹ allows slabs to be designed by any procedure that satisfies equilibrium and geometric compatibility, provided that every section has a design strength at least equal to the required strength and serviceability conditions are fulfilled. In traditional strip methods that consider independent spans in two orthogonal directions, there is no defined means of ensuring deformation compatibility or equilibrium between the two directions. In contrast, finite element analysis (FEA) automatically provides deformation compatibility (for considered degrees-of-freedom between elements) and a full equilibrium load path. Designers using FEA, however, have often blindly ignored twisting moments, an assumption that may be unconservative where twists are high, such as in the corner regions of slabs.² To provide designers with guidance related to this issue, available methods for explicitly incorporating twisting moments in the design of slabs based on FEA results are discussed in this article.

FINITE ELEMENT ANALYSIS OF SLABS

The finite element method essentially approximates slab behavior by subdividing the plate continuum into a mesh of discrete finite elements. Plate or shell elements are typically employed to represent the behavior of slabs by deformations at the midsurface. Figure 1 shows the shear forces (q_x and q_y) and bending (m_x and m_y) and twisting (m_{xy}) moments resulting from transverse loading q for an infinitesimal plate element. Also, Fig. 2(a) shows a typical triangular plate element with three degrees-of-freedom at each node (w_N is the out-of-plane translation, and

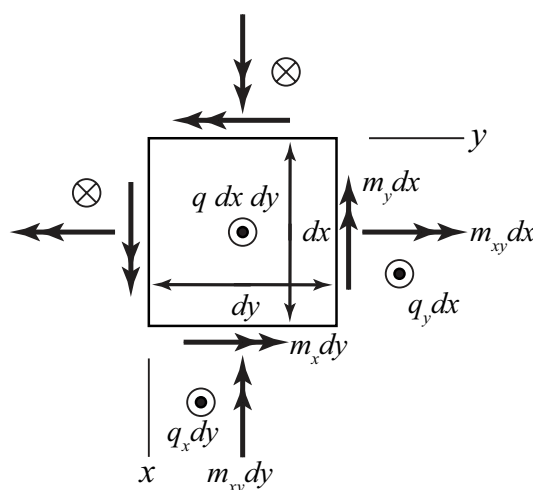


Fig. 1: An infinitesimal plate element shown with resulting shear forces and moments due to transverse loading (\otimes and \odot indicate shear forces into the plane and out of the plane, respectively)

$\partial w_N / \partial x$ and $\partial w_N / \partial y$ are the two rotations about the y - and x -axes, respectively, at the N th node) and corresponding element nodal forces. Nodal displacements for a plate element are acquired by solving the global structure equilibrium equation. Element nodal displacements can then be used to compute internal forces needed for slab design, usually based on one of two approaches: moment fields using moment-curvature relations (the classical approach) or element nodal forces using the element stiffness matrix.

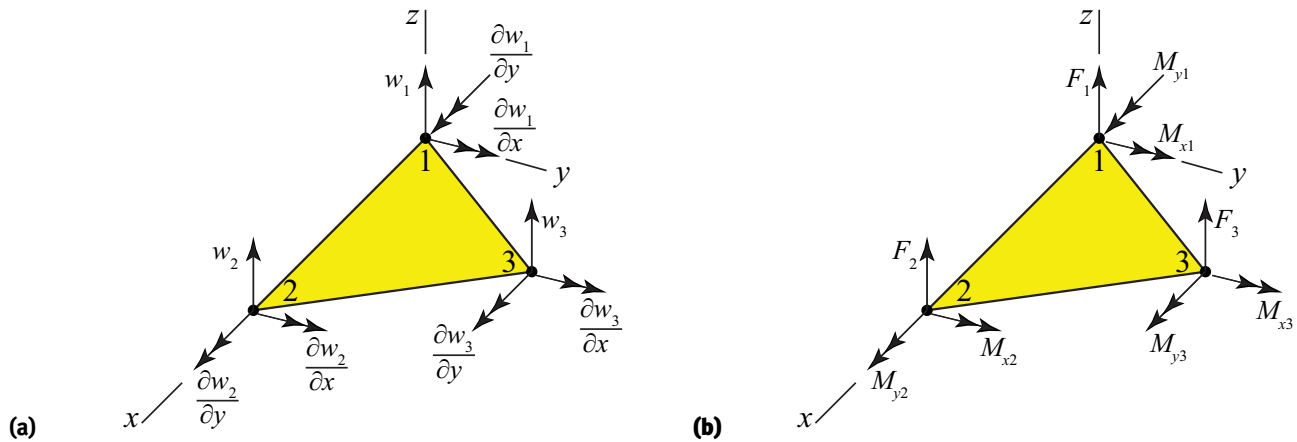


Fig. 2: A typical triangular plate element used to model slabs: (a) three degrees of freedom at each node; and (b) element nodal forces (forces shown with single arrows, moments shown with double arrows)

Computation of moment fields

Assuming the slab is relatively thin so that transverse shear strains are negligible, bending and twisting moment fields for each isotropic plate element, as an example, can be computed as follows³

$$\begin{Bmatrix} m_x \\ m_y \\ m_{xy} \end{Bmatrix} = - \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & (1-\nu)D/2 \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} = -[D_k][B]\{d_e\} \quad (1)$$

where $[D_k]$ is the moment-curvature relationship expressed in terms of the flexural rigidity of the plate D and Poisson's ratio ν , $[B]$ is the strain-displacement matrix, $\{d_e\}$ is the element nodal displacement vector, and $[B]\{d_e\}$ represents the plate curvatures. Note that resulting moments m_x , m_y , and m_{xy} have units of moment per unit length.

An extremely fine mesh is required to accurately compute bending and twisting moments due to the generic approximation of plate curvatures. Additionally, moments may exhibit singularities arising from concentrated loads or reactions at locations such as slab-to-column connections.

Computation of element nodal forces

An alternative approach used for slab design involves element nodal forces computed at the nodes of each element. Conceptually, element nodal forces for a plate element bear a similarity to member end forces for a frame element. The element nodal forces $\{F_e\}$ are computed based on the element equilibrium equation

$$\{F_e\} = [K_e]\{d_e\} \quad (2)$$

where $[K_e]$ is the element stiffness matrix. (Note that the equivalent nodal load vector is not considered for simplicity.) In contrast to moments computed based on the moment-curvature relations, $\{F_e\}$ contains forces and moments directly corresponding to each degree-of-freedom of the plate element, as shown in Fig. 2(b). Using this approach, equilibrium with externally applied nodal loads is guaranteed, such that element nodal forces are highly accurate even for a coarse finite element mesh.

Total moment on design sections

Ultimately, an engineer must use the results of FEA to find the total moment acting on the design section of a selected design strip, as illustrated in Fig. 3. Using the classical approach, moments varying across the design section must be integrated on a tributary area basis, after the effect of twist has been incorporated. Because the moment values being integrated are only approximate due to the inaccuracy of the curvatures, the total moment calculated is not guaranteed to be in equilibrium with the applied loads.

Element nodal forces, in contrast, can be employed to compute section forces by equilibrium consideration of the entire design section. This approach transforms all element nodal forces acting at nodes on the design section into resultant forces and moments acting at the centroid, as shown in Fig. 4; shell elements with six degrees of freedom at each node are used for illustration. In this case, the components requiring special attention for slab design include the total bending moment M as well as the total torsion T .

SLAB TWISTING MOMENTS IN ACI 318

ACI 318-08¹ does not explicitly address twisting moments or FEA in the sections dealing with slab design, but Section 19.4 on shell reinforcement requires that

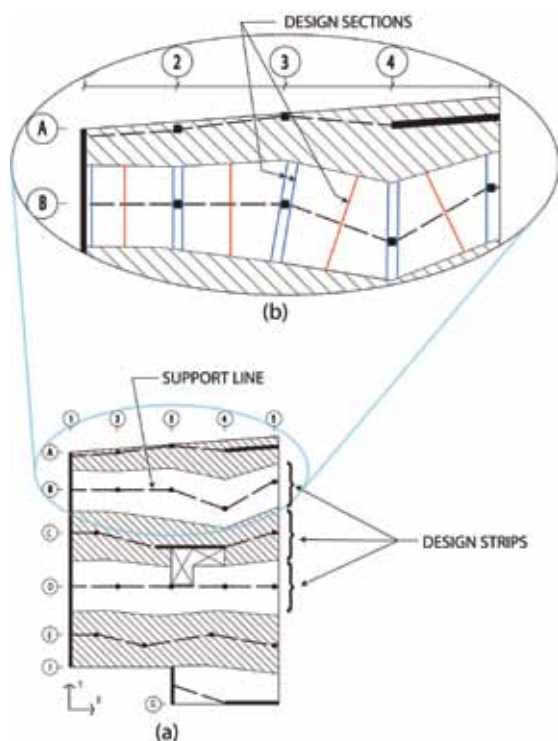


Fig. 3: Design strips for an irregular floor slab: (a) support lines and design strips; and (b) design sections (indicated by red and blue lines) for design strip along Column Row B (adapted from Reference 4)

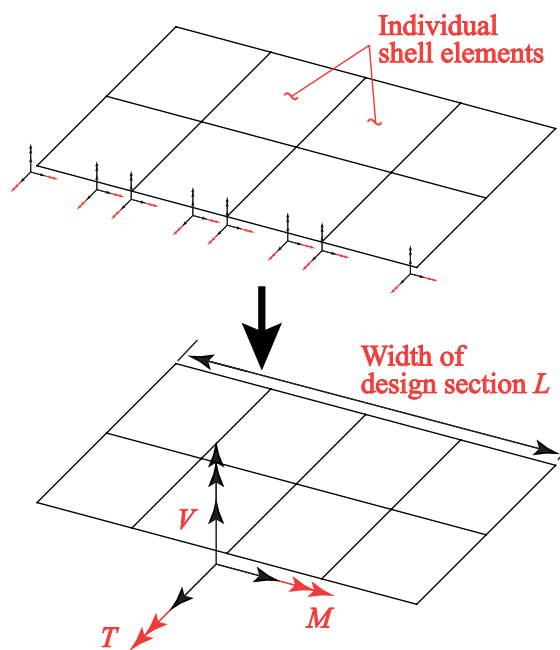


Fig. 4: Element nodal forces from individual shell elements along a design section can be transformed into equivalent total forces acting on the design section (forces shown with single arrows, moments shown with double arrows)

reinforcement be designed to resist “tension from bending and twisting moments.”

The lack of a provision for twisting moments in the design of slabs does not necessarily make the code unconservative. Two slab design procedures presented in ACI 318—the Direct Design Method and the Equivalent Frame Method—both have roots in traditional frame analysis. Slabs designed based on these methods have a history of good performance and adequate strength, especially for usual proportions with supports arranged in a regular grid pattern. The Code provisions, however, do not necessarily guarantee a conservative slab design when significant twisting moments exist.

DESIGN USING CLASSICAL APPROACH

Several slab design methods have been proposed based on bending and twisting moments computed using the moment-curvature relations. In most designs, slab reinforcement is placed along two orthogonal directions. The fundamental premise of this suite of methods is to augment the bending moments in these orthogonal directions in consideration of the twisting moments prior

to integrating the total moment over the design section.

Wood and Armer method

Wood and Armer⁵ proposed one of the most popular design methods that explicitly incorporate twisting moments in slab design. This method was developed by considering the normal moment yield criterion (Johansen’s yield criterion)² aiming to prevent yielding in all directions. At any point in the slab, the moment normal to a direction, resulting due to design moments m_x , m_y , and m_{xy} , must not exceed the ultimate normal resisting moment in that direction. The ultimate normal resisting moment is typically provided by ultimate resisting moments m_{ux} and m_{uy} related to the reinforcement in the x - and y -directions. Design rules based on the Wood and Armer method can be stated as follows²

$$m_{ux} = m_x \pm |m_{xy}| \quad (3a)$$

$$m_{uy} = m_y \pm |m_{xy}| \quad (3b)$$

where all plus signs apply only to bottom reinforcement, and all minus signs apply only to top reinforcement (m_{ux} and m_{uy} will be negative for tension in the top reinforcement).

For bottom reinforcement, when either m_{ux} or m_{uy} is found to be negative, the negative moment is assigned a value of zero and the other moment is modified as follows

$$m_{ux} = m_x \pm \left| \frac{m_{xy}^2}{m_y} \right|, \text{ with } m_{uy} = 0 \quad (4a)$$

or

$$m_{uy} = m_y \pm \left| \frac{m_{xy}^2}{m_x} \right|, \text{ with } m_{ux} = 0 \quad (4b)$$

where all plus signs apply only to bottom reinforcement. Similarly, for top reinforcement, when either m_{ux} or m_{uy} from Eq. (3a) or (3b) is found to be positive, the moment is assigned a value of zero and the moment in the perpendicular direction is determined using Eq. (4a) or (4b), in which all minus signs apply only to top reinforcement. For these cases, the factors $|m_{xy}/m_y|$ or $|m_{xy}/m_x|$ are always less than 1, producing smaller moments than given by Eq. (3a) or (3b).

The Wood and Armer approach is unconditionally stable with respect to the torsion component. Several studies,⁶ however, have indicated that this method may produce unconservative results for slabs with high reinforcement ratios (approximately more than 0.75%) at regions of significant twisting moments, especially near restrained slab corners. Also, it is noted that this method is not applicable to flat slabs with beams or drop panels.

Canadian Code

Section 13.6.4 of the Canadian Code⁷ implements a simplified version of the Wood and Armer method. When the assumed-to-be-positive design moment is found to be negative (or vice versa), that moment is taken as zero, but the moment in the other direction is still calculated using Eq. (3a) or (3b). Therefore, this Canadian Code approach generally leads to solutions that are more conservative than the Wood and Armer approach.

Eurocode 2

The current version of Eurocode 2⁸ no longer explicitly specifies what to do with twisting moments when designing slabs. The previous version of Eurocode 2,⁹ however, specified design moment equations similar to and even more general than those proposed by Wood.⁵ Section A.2.8 (4) of this Code allowed various sets of coefficients for the torsional component $|m_{xy}|$ that satisfy a certain relationship and produce design moments no more than twice and no less than half of the values determined by the Wood and Armer approach. For example, bottom reinforcement was allowed to be designed using the following equations with different γ values

$$m_{ux} = m_x + \gamma |m_{xy}| \quad (5a)$$

$$m_{uy} = m_y + \frac{1}{\gamma} |m_{xy}| \quad (5b)$$

Although this Code intended to allow more liberal reinforcement arrangements, it's noted that the resisting moments calculated with γ equal to one are the most economical solutions.²

CEB-FIP Model Code

According to Sections 6.4.1 and 6.5.4 of CEB-FIP Model Code,¹⁰ a slab section is divided into three layers, each with 1/3 of the slab thickness. Then, bending moments are decomposed into a couple of tensile and compressive normal forces and twisting moments are decomposed into a couple of in-plane shear forces acting at top and bottom layers of the slab. The sandwiched inner layer is used for transverse shear. The internal lever arm between the top and bottom shear forces due to the twisting moment may be taken as 2/3 the overall slab thickness. These normal and shear forces due to moments are combined with membrane (in-plane) forces. Finally, slab reinforcement is proportioned to satisfy equilibrium conditions at each top and bottom layer in the two orthogonal directions. More detailed derivations for this method can be found elsewhere.¹¹

DESIGN USING ELEMENT NODAL FORCES

Slab design methods based on element nodal forces have been widely implemented in FEA software. These methods are attractive because results are relatively accurate even for very coarse meshes, can be used for slabs containing beams or drop panels, and are easily extended to design prestressed or post-tensioned floors. In this approach, the critical question facing engineers is how to account for the total torsion T computed across the design section, which is separate from the total primary bending moment M , as illustrated in Fig. 4.

Principally aligned design sections

The effect of twisting moments can be ignored when design sections are aligned with the principal bending directions at all locations, as the twist on the design section vanishes in this case. A rule of thumb that is sometimes used is that torsion and twist can be ignored if T is less than 10% of M .¹² If the torsion is greater than this value, then neglecting the torsion and twist effects may lead to unconservative results.

Many commercial FEA programs are capable of plotting vectors corresponding to the directions of principal bending; and after reviewing such results, design sections can be chosen to be orthogonal to these principal bending directions. When these directions do not match with desired reinforcement directions, the components of required reinforcement are determined first with respect to the principal bending directions and

then transformed via a simple change of axes into the desired orientations.

Bending modified by torsion

One approach to incorporating T is to directly combine M with $\pm T$, similar to the Wood and Armer approach. From a mechanics standpoint, however, this concept differs from the Wood and Armer method in that the moments are resultants from the entire design section, not an infinitesimal plate element. In addition, this approach can lead to highly uneconomical designs when T is significant compared with M .

"Beam" torsion

Another approach that has been implemented in commercial FEA software is to account for T according to building code provisions for beams in torsion. This approach resists torsion in the slab by hoop stresses, and reinforcement is proportioned to satisfy this load path.

Torsion resolved into transverse shear

The total torsion T in the design section can be decomposed into linearly varying transverse shear per unit length with a maximum value equal to $6T/L^2$, where L is the width of the design section. By multiplying this maximum shear by the width of the design section, an equivalent (but quite conservative) resultant shear force due to torsion, which is equal to $6T/L$, can be considered during the shear design of the section.

DESIGN USING "TWIST-FREE" ANALYSIS

Slab analysis and design software that is capable of analyzing orthotropic slabs (with different properties in two orthogonal directions) typically allows engineers to separately modify the stiffness corresponding to each of the bending moments about the two orthogonal axes and the twisting moment.

Setting the twist stiffness to zero leads to an analytical solution (and a load path) in which all of the twisting moments are zero. The behavior and load path of a twist-free slab is somewhat similar to that of a grillage of beams.

Bending and twisting moments are interrelated via a Mohr's circle equilibrium constraint. Because of this constraint, the twist stiffness can only be set to zero about a single set of perpendicular axes at each location. The most practical axes for this purpose are the axes of the design cross sections (perpendicular to the reinforcement). In a typical slab that is reinforced in only two perpendicular directions, the entire slab can be given

the same modified stiffness. For slabs where the reinforcement directions vary from region to region, multiple stiffness modifications will be required.

Twist-free slab analyses will result in larger calculated deflections than those predicted by analyses with non-zero twist stiffness. This is to be expected as the load path with non-zero twist stiffness is more efficient in terms of elastic energy than one with zero twist stiffness. Comparing slab deflections computed by analyses with and without twist stiffness is a quick means of gauging how much loads will be redistributed to mobilize the twist-free load path. If the deflections from the twist-free analysis are significantly greater than

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those from the analysis with twist stiffness (for example, by a factor of 2), it should be investigated whether excessive twist-related cracking might occur.

CONCLUSIONS

When designing concrete slabs, neglecting twisting moments that are part of the equilibrium load path may result in unconservative solutions, especially when supports are arranged in an irregular pattern. There are multiple tools available in commercial FEA-based slab design software that account for twisting moments. Unfortunately, there is not a single method for dealing with twisting moments from FEA that works best for all situations and can be easily incorporated in design codes. The engineer must therefore understand the principles of the available methods and choose a reasonable method for the slab being designed.

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Selected for reader interest by the editors after independent expert evaluation and recommendation.



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